A NEW CUMULANT BASED INVERSE FILTERING ALGORITHM FOR IDENTIFICATION AND DECONVOLUTION OF NONMINIMUM-PHASE SYSTEMS

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ABSTRACT

In this paper, we propose a new cumulant based inverse filtering algorithm for the identification and deconvolution of linear time-invariant (LTI) nonminimum-phase systems with only non-Gaussian output measurements contaminated by Gaussian noise. Some simulation results are provided to demonstrate that the proposed algorithm works well.

1. INTRODUCTION

The identification of a linear time-invariant (LTI) system $h(k)$ from noisy output $x(k)$ based on the following convolutional model:

$$x(k) = y(k) + n(k) = u(k) * h(k) + n(k) \qquad (1)$$

is very important in many signal processing areas such as seismic deconvolution, channel equalization, radar, sonar, speech processing and image processing. Recently, cumulant (higher order statistics) based identification [1-5] of nonminimum-phase LTI systems with only non-Gaussian output measurements has drawn extensive attention in the previous signal processing areas because cumulants, which are blind to any kind of a Gaussian process, not only extract the amplitude information but also the phase information of $h(k)$, meanwhile they are inherently immune from Gaussian measurement noise $n(k)$.

Higher order statistics based inverse filter criteria [6-9] have been used to estimate $h(k)$. In this paper, we propose a new cumulant based inverse filtering algorithm for the identification of $h(k)$ as well as the estimation of the desired signal $u(k)$.

2. A NEW INVERSE FILTERING ALGORITHM

Assume that data $x(k), k = 0, 1, \cdots, N-1$ were generated from the model given by (1). The new inverse filtering algorithm to be presented below are based on the following modeling assumptions:

(A1) The system $h(k)$ is causal and exponentially stable; it can be minimum-phase or nonminimum-phase.

(A2) The input $u(k)$ is real, zero-mean, stationary, independent identically distributed (*i.i.d.*), non-Gaussian with Mth-order cumulant γ_M.

(A3) The measurement noise $n(k)$ is Gaussian which can be white or colored with unknown statistics.

(A4) The input $u(k)$ is independent of $n(k)$.

Next, we present the following theorem on which the new inverse filtering algorithm is based.

Theorem 1. Assume that $x(k)$ was generated from (1) under the previous assumptions (A1) through (A4). Let $e(k)$ be the output of a stable LTI filter $v(k)$ with the input $x(k)$. Let $\hat{v}(k)$ denote the optimum $v(k)$ based on the following criterion

$$J(v(k)) = \frac{1}{C^2_{M,e}(\underline{0})} \sum_{\underline{k}} C^2_{M,e}(\underline{k}) \geq J(\hat{v}(k)) \geq 1 \qquad (2)$$

where $M \geq 3$, $\underline{k} = (k_1, \cdots, k_{M-1})$ and $C_{M,e}(\underline{k})$ is the Mth-order cumulant function of $e(k)$. Then $\hat{v}(k) * h(k) = \alpha\delta(k-l)$ and the associated $\hat{e}(k) = \alpha u(k-l)$ where l is an integer and $\alpha \neq 0$.

Proof: The signal $e(k) = x(k) * v(k) = \varepsilon(k) + \eta(k)$ where $\varepsilon(k) = y(k) * v(k) = u(k) * h(k) * v(k)$ and $\eta(k) = n(k) * v(k)$. Thus $C_{M,e}(\underline{k}) = C_{M,\varepsilon}(\underline{k})$ since $\eta(k)$ is Gaussian. It is trivial to see that $J \geq 1$. That $J = 1$ happens when $C_{M,e}(\underline{k}) = C_{M,\varepsilon}(\underline{k}) = 0$ for $\forall \underline{k} \neq \underline{0}$, from which one can easily infer that $\hat{v}(k) * h(k) = \alpha\delta(k-l)$ and the associated $\hat{e}(k) = \alpha u(k-l)$ where l is an integer and $\alpha \neq 0$. Q.E.D.

Let $v(k)$ be a finite impulse response (FIR) filter of order L and $e(k)$ be the output of $v(k)$ in response of the input $x(k)$, i.e.,

$$\begin{aligned} e(k) &= \sum_{j=0}^{L} v(j)\, x(k-j) \\ &= (x(k), x(k-1), ..., x(k-L))\ \underline{v}_L \end{aligned} \quad (3)$$

where

$$\underline{v}_L = (v(0), v(1), ..., v(L))^T. \quad (4)$$

Based on Theorem 1, we estimate the inverse filter $v(k)$ of $h(k)$ by minimizing the following objective function

$$J(L) = \frac{D(L)}{\hat{C}^2_{M,e}(\underline{0})} \quad (5)$$

where

$$D(L) = \sum_{\underline{k}} \hat{C}^2_{M,e}(\underline{k}) \quad (6)$$

in which $\hat{C}_{M,e}(\underline{k})$ is the biased Mth-order sample cumulant function of $e(k)$. For example, for $M = 3$,

$$\hat{C}_{3,e}(k_1, k_2) = \frac{1}{N} \sum_{k=K_1}^{k=K_2} e(k)e(k+k_1)e(k+k_2) \quad (7)$$

where $K_1 = max(0, -k_1, -k_2)$ and $K_2 = min(N-1, N-1-k_1, N-1-k_2)$. Moreover, since $J(L)$ is a highly nonlinear function of $\underline{v}_L$, it is almost impossible to find a closed-form solution for the optimum $\hat{\underline{v}}_L$. Instead, we resort to an iterative numerical optimization method to search for the desired $\hat{\underline{v}}_L$. The new cumulant based inverse filtering algorithm is as follows:

(s1) Let $L = 0$ and compute $J(L)$ with $\hat{\underline{v}}_L = 1$ (i.e., $\hat{v}(k) = \delta(k)$).

(s2) Set $L = L + 1$.

(s3) Search for the $J_{min}(L)$ (minimum of $J(L)$) and the associated $\hat{\underline{v}}_L$ by a Newton-Raphson type iterative algorithm.

(s4) If $J_{min}(L)$ converges, then stop; otherwise go to (s2).

The Newton-Raphson type iterative algorithm used in (s3) for the case of $M = 3$ is summarized in *Appendix A*. Three remarks regarding the proposed algorithm are noteworthy as follows:

(R1) The $D(L)$ in $J(L)$ (see (5) and (6)) can only be calculated over a finite (M-1)-dimensional region. We calculate $D(L)$ over the finite domain of support $F(q)$ of the Mth-order cumulant function of a non-Gaussian $MA(q)$ process which can be viewed as an approximation to the non-Gaussian linear process $e(k)$. For instance, for $M = 3$, the finite region $F(q)$ is the following hexagonal region:

$$F(q) = \left\{ \underline{k} \,\middle|\, |k_1| \le q, |k_2| \le q, |k_1 - k_2| \le q \right\}. \quad (8)$$

(R2) The initial guess $\hat{\underline{v}}_L^T(0) = (\hat{\underline{v}}_{L-1}^T, 0)$ for $L \ge 2$, where $\hat{\underline{v}}_{L-1}$ is the optimum estimate of $\underline{v}_{L-1}$ associated with $J_{min}(L-1)$, is used to initialize the iterative algorithm in (s3) except that the initial guess $\hat{\underline{v}}_1^T(0) = (0.7071, 0.7071)$. In each iteration of the iterative algorithm, the objective function $J(L)$ is guaranteed to decrease, meanwhile $\hat{\underline{v}}_L(i)$ is normalized such that $||\hat{\underline{v}}_L(i)|| = 1$. Therefore, $J_{min}(L)$ decreases monotonically with L which together with the fact of $J_{min}(L) \ge 1$ guarantees the convergence of the proposed algorithm.

(R3) The proposed inverse filtering algorithm can be used for any $M \ge 3$ (order of cumulants) as long as the Mth-order cumulant γ_M of the driving input $u(k)$ is not equal to zero.

3. SIMULATION RESULTS

In this section, we show two simulation examples to support that the proposed algorithm works well. The driving noise $u(k)$ used was a zero-mean, Exponentially distributed random sequence with variance $\sigma_u^2 = 1$ and skewness $\gamma_3 = 2$. We passed this sequence through a selected LTI system $H(z)$ to obtain the noise-free output signal $y(k)$ and then added a zero-mean white Gaussian noise sequence $n(k)$ to $y(k)$ to form the synthetic noisy data $x(k)$ for signal-to-noise ratio (SNR) equal to 10. The order of cumulants used was $M = 3$ and the length of data was N=1024. Mean and standard deviation of $\hat{v}(k)'s$ were calculated from 30 independent estimates obtained by the proposed inverse filtering algorithm. Now, let us turn to Example 1.

Example 1: AR process of known order

A second-order AR system

$$H(z) = 1/A(z) = 1/(1 + 0.7z^{-1} + 0.1z^{-2})$$

was used. For each run, the estimate $\hat{\underline{v}}_2$ was obtained only through the procedure (s3) of the proposed inverse filtering algorithm where $L = 2, q = 5$ in the calculation of $D(L)$ and the initial guess for $\underline{v}_2^T$ was (0.5774, 0.5774, 0.5774). The simulation results for J_{min} and the associated optimum $\hat{v}(k)$ are shown in Table 1, from which one can see that $\hat{v}(k)$ approximates well the impulse response $a(k)$ of the inverse filter $1/H(z) = A(z)$, except for a scale factor 0.8133 ($\cong V(z)/A(z) = 0.8165$). The simulation results are consistent with Theorem 1 and support that the proposed algorithm works well.

Table 1. Simulation results for Example 1.

a(0)=1, a(1)=0.7, a(2)=0.1, N=1024, SNR=10, 30 independent runs.	
J_{min}	1.0569±0.0294
$\hat{v}(0)$	0.8133±0.0194
$\hat{v}(1)$	0.5744±0.0268
$\hat{v}(2)$	0.0837±0.0261

Example 2: ARMA process

A nonminimum-phase ARMA model

$$H(z) = \frac{1 - 2.7z^{-1} + 0.5z^{-2}}{1 + 0.1z^{-1} - 0.12z^{-2}}$$

was used for this example. The first estimate $\hat{v}(k)$ was obtained through the previous procedure (s1) through (s4) until $L = 16$ where $q = 7$ in the calculation of $D(L)$. For the other 29 runs, with the first estimate as the initial guess of $v(k)$, we only went through the procedure (s3) with $L = 16$ and $q = 7$ to obtain the optimum $\hat{v}(k)$. The thirty estimates of $v(k)$ are shown in Fig. 1 where we have artificially compensated for scale factor and time-delay so that $\hat{v}(k)'s$ can be clearly compared with the impulse response of $1/H(z)$. Note, from Fig. 1(a), that different kinds of lines are used to make each single $\hat{v}(k)$ discernible from other $\hat{v}(k)'s$. One can also see, from Fig. 1(b), that the mean (dashed line) of the thirty estimates of $v(k)$ approximates well the impulse response (solid line) of the inverse filter $1/H(z)$. Again, these simulation results are consistent with Theorem 1 and justify that the proposed cumulant based inverse filtering algorithm works well.

4. CONCLUSIONS

In this paper, we have presented a new cumulant based inverse filtering algorithm based on Theorem 1 for the identification and deconvolution of a LTI nonminimum-phase system $H(z)$ with only non-Gaussian output measurements contaminated by Gaussian noise. The inverse filter assumed to be a FIR filter of order L is estimated through the identification procedure (s1)-(s4) of the proposed inverse filtering algorithm described in Section 2. We also guarantee the convergence of the proposed inverse filtering algorithm (see (R2)). Surely, the parameter L must be set large enough for good approximation of $\hat{V}(z)$ to the inverse filter $1/H(z)$. Once the inverse filter $\hat{V}(z) = 1/\hat{H}(z)$ is obtained, $\hat{H}(z)$ can be easily obtained from $\hat{V}(z)$. We also showed some simulation results which not only are consistent with Theorem 1 but also support the good performance of the proposed cumulant based inverse filtering algorithm.

APPENDIX A

Newton-Raphson Type Algorithm Used in (s3)

The Newton-Raphson type algorithm updates $\hat{\underline{v}}_L(i)$ at the ith iteration by

$$\hat{\underline{v}}_L(i) = \hat{\underline{v}}_L(i-1) - \rho[Tr(H_{i-1})]^{-1}\underline{g}_{i-1} \quad (9)$$

where $0 < \rho \leq 1$, $Tr(H_{i-1})$ denotes the trace of H_{i-1}, and $\underline{g}_{i-1}$ as well as H_{i-1} denote the gradient and the Hessian matrix for $\underline{v}_L = \hat{\underline{v}}_L(i-1)$, respectively, as follows:

$$\underline{g}_{i-1} = \left.\frac{\partial J(L)}{\partial \underline{v}_L}\right|_{\underline{v}_L = \hat{\underline{v}}_L(i-1)} \quad (10)$$

$$H_{i-1} = \left.\frac{\partial^2 J(L)}{\partial \underline{v}_L^2}\right|_{\underline{v}_L = \hat{\underline{v}}_L(i-1)} \quad (11)$$

At each iteration, updating $\hat{\underline{v}}_L$ by (9) with $\rho = 1$ normally leads to the decrease of $J(L)$; otherwise, a smaller ρ must be considered.

Assume that M=3. Next, we present how to compute $\partial\hat{C}_{3,e}(\underline{k})/\partial\underline{v}_L$ and $\partial^2\hat{C}_{3,e}(\underline{k})/\partial\underline{v}_L^2$ which are needed for computing $\underline{g}_{i-1}$ and H_{i-1}. Taking the partial derivative of (3) with respect to $v(m)$, $m = 0, 1, \cdots, L$, we find

$$\frac{\partial e(k)}{\partial v(m)} = x(k-m), \; m = 0, 1, \cdots, L. \quad (12)$$

Let

$$C_{bcd}(i,j,\ell) = \frac{1}{N}\sum_{k=K_1}^{k=K_2} b(k+i)c(k+j)d(k+\ell). \quad (13)$$

From (7), (12) and (13), we have

$$\frac{\partial\hat{C}_{3,e}(\underline{k})}{\partial v(m)} = C_{xee}(-m, k_1, k_2)$$
$$+ C_{xee}(k_1 - m, 0, k_2) + C_{xee}(k_2 - m, 0, k_1). \quad (14)$$

Further taking the partial derivative of (14) with respect to $v(s)$, $s = 0, 1, \cdots, L$, we obtain

$$\frac{\partial^2\hat{C}_{3,e}(\underline{k})}{\partial v(s)\partial v(m)} = C_{xxe}(-m, k_1 - s, k_2)$$
$$+ C_{xxe}(-s, k_1 - m, k_2) + C_{xex}(-m, k_1, k_2 - s)$$
$$+ C_{xex}(-s, k_1, k_2 - m) + C_{exx}(0, k_1 - m, k_2 - s)$$
$$+ C_{exx}(0, k_1 - s, k_2 - m). \quad (15)$$

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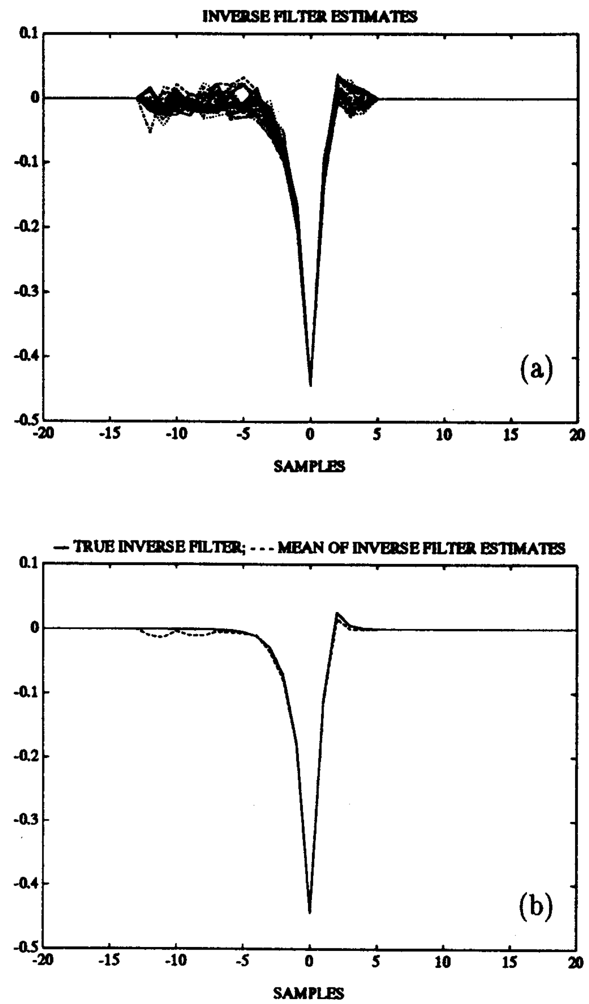

Figure 1: Simulation results for Example 2. (a) Thirty estimates of $v(k)$ and (b) impulse response (solid line) of the true inverse filter $1/H(z)$ and mean (dashed line) of the thirty estimates of $v(k)$.